M.Sc. DEGREE EXAMINATION — MARCH/APRIL 2021

Part I – Previous

Branch — Mathematics

Paper I — ALGEBRA

(Revised Regulation from 2009-2010)

(Regular/Supplementary)

Max. Marks : 20

PART – A

Answer any FOUR questions. Each question carries 5 marks.

(Marks : 4 × 5 = 20)

1. Prove that every group of order p^2 (p prime) is abelian.

2. Prove that

   (a) Every finite group has composition series and

   (b) Any two composition series of a finite group an equivalent.

3. Prove that a nonempty subset S of ring R is a subring if and only if for all $Q,b \in S$, $a-b \in S$ and $a.b \in s$.

4. If a fermutation $\sigma \in S_n$ is a product of r transpositions and also a product of s transposition, Then prove that r and s are either both even or r odd.

5. Let $\{N_i\}_{i \in I}$ be a family of R sub modulus of an R-module M. Then prove that $\bigcap_{iGI} N_i$ is also an R-submodule.

6. State and prove the Cauchy's Theorem for abelian groups.

7. Prove that any totally ordered set is distributive lattice.

8. Prove that for any set $S, P(S)$, the power set of S is a complete lattice.

[P.T.O.]

UNIT – IV

15. (a) Let $(B,\vee,\wedge,0,1)$ be a Boolean algebra. Define operation on B by

    (i) $a+b=(a\wedge b')\vee(a'\wedge b)\vee(a'\wedge b)$,

    (ii) $a\wedge b=a,b$. Then prove that $(B,+,\bullet)$ in a Boolean ring.

    (b) Prove that a partiâlly ordered set with a least element 0 such that every non-empty subset has a least upper bound is a complete lattice.

Or

16. (a) Show that a lattice of invariant sub groups of any group is modular.

    (b) Prove that the set of all normal subgroups of a group G is modular lattice under the operations defined by

    (i) $H_1\wedge H_2=H_1\wedge H_2$

    (ii) $H_1\vee H_2=H_1H_2$ for all normal subgroups H_1,H_2 of G.

M.Sc. DEGREE EXAMINATION — MARCH/APRIL 2021

Part I – Previous

Branch – Mathematics

Paper II— REAL ANALYSIS

(Revised Regulations from 2009–2010)

Max. Marks : 20

PART A

Answer any FOUR questions.

Each question carries 5 marks.

(Marks: 4 × 5 = 20)

1. Prove that every infinite subset of a countable set A is countable.

2. Suppose $f \geq 0$, f is continuous on $[a, b]$, then prove that $f(x) = 0$ for all $x \in [a, b]$.

3. Explain in detail what is R-S intergrability.

4. The sequence of a function $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $E > 0$, there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E \left|f_n(x) - f_m(x)\right| \leq E$.

5. Prove that why uniformly convergent sequence of bounded function is uniformly bounded.

6. State and prove Libesquis monotone converges theorem.

7. Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges if and only if $n < 1$.

8. Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

[P.T.O.]

12. Let $f(x) = \begin{cases} 0 & if\ x < \frac{1}{n+1} \\ \sin^2 \frac{\pi}{x} & if \frac{1}{n+1} \le x \le \frac{1}{n} \\ 0 & if \frac{1}{n} < x \end{cases}$

Show that $\{f_n\}$ converges to a continuous function, but not uniformly. Use the series ε f_n to show that absolute convergence, even for all x, does not imply uniform convergence.

UNIT – III

13. (a) Suppose f is measurable and non-negative on x. For $A \in M$, define $\phi(A) - \int_A f\ d\mu$ prove that ϕ is continuously additive on M.

(b) With the usual notation, prove that $M(\mu)$ is a σ-ring and μ^* is countably additive on $M(\mu)$.

Or

14. (a) State and prove Fatou's theorem.

(b) If k is a compact metric space if $f_n \in l(k)$ for $n = 1,2,3,....$ and if $\{f_n\}$ converges uniformly on k, then $\{f_n\}$ is equii continuous on k.

UNIT – IV

15. (a) Find the Fourier Sine and Cosine series for $f(x) = (\pi - x)$ in $0 < x < \pi$.

(b) If f_x and f_y on both differentiable at a point (a,b) of the domain function f then show that $f_{xy}(a,b) = f_{yx}(a,b)$.

Or

16. (a) Find the extreme values of the function $f(x,y,z) = (x+y+z)^3 - 3(x+y+z) - 2uxyz + 8$.

(b) State and prove Young's theorem.

M.Sc. DEGREE EXAMINATION, APRIL/MAY -2022

PART I - PREVIOUS

BRANCH : MATHEMATICS

Paper - III : DIFFEENTIAL EQUATIONS

(For D.D.E Students)

(Revised Regulations from 2010-2011)

(Regular/Supplementary)

Max. Marks : 20

SECTION - A

Answer any **Four** questions. Each question carries 5 marks. **(4×5=20)**

1. Prove that there are three linearly independent solutions of the third order equation. $x^{111}+b_1(t)x^{11}+b_2(t)x^1+b_3(t)x=0$, $x \in I$ where I is an interval of the real line $\mathbb{R}$, where b_1, b_2 and b_3 are functions defined and continuous on I. **(5)**

2. The motion of a simple pendulum is $x^{11}(t)+k\sin x(t)$, where k is a constant. Find power series solution of this equation that satisfies the initial conditions $x(0)=\frac{\pi}{6}$ and $x^1(0)=0$. **(5)**

3. Let A (t) be an n×n matrix which is continuous on I. Suppose a matrix Φ satisfies $X^1 = A(t)X$. Then prove that det Φ satisfies the first order equation $(\det \Phi)^1 = (t\,A)(\det \Phi)$. **(5)**

4. Let f(t,x) be a continuous function defined over a rectangle $R=\{(t,x):|t-t_0|\le p, |x-x_0|\le q\}$. Here p, q are some positive real numbers. Let $\frac{\partial f}{\partial x}$ be defined and continuous on R. Then prove that f (t, x) satisfies the Lipchitz condition in R. **(5)**

5. Let $v, w \in C^1\{[t_0, t_0+h], R\}$ be lower and upper solutions of $x^1 = f(t,x), x(t_0)=x_0$ respective where $f \in C[D,\mathbb{R}]$, where D is an open connected set in $\mathbb{R}^2$ and $(t_0, x_0) \in D$. Suppose that, for $x \ge y$, f satisfies the inequality $f(t,x)-f(t,y)\le L(x-y)$, where L is a positive constant. Then prove that $v(t_0)\le w(t_0)$ implies that $v(t)\le w(t)$, $t\in[t_0, t_0+h]$ **(5)**

b) Let $x(t)=x(t;t_0,x_0)$ and $x^*(t)=x(t,t_0^*,x_0^*)$ be solutions of the I VPS $x^1=f(t,x),x(t_0)=x_0$; and $x^1=f(t,x),x(t_0^*)=x_0^*$ respectively on an interval $a\le t\le b$. Let $(t,x(t)),(t,x^*(t))$ lie in a domain D for $a\le t\le b$. Further, let $f\in Lip(D,k)$ be founded by L in D. Then, prove that for any $\in>0$, there exists $\delta=\delta(\in)>0$ such that $|x(t)-x^*(t)|<\in$, $a\le t\le b$ whenever $|t_0-t_0^*|<\delta$ and $|x_0-x_0^*|<\delta$. **(7)**

Unit-III

13. a) State and prove Bihari's inequality. **(7)**

b) State and prove Alekseev's formula. **(8)**

(OR)

14. a) Let y and z be linearly independent solutions of $L(x)=(px^1)^1+qx=0$ where p,p^1 & q are real valued continuous functions on $a\le t\le b$. Define Green's function by

$$G(s,t)=\begin{cases}\dfrac{-y(t)z(s)}{A} & if \quad t\le s\\ \dfrac{-y(s)z(t)}{A} & if \quad t\ge s\end{cases},$$

where $A=p(t)[y(t)z^1(t)-y^1(t)z(t)]$, $t\in[a,b]$, a nonzero constant. Then prove that x (t) is a solution of $L(x)+f(t)=0$, $a\le t\le b$, with boundary conditions

$m_1x(a)+m_2x^1(a)=0$

$m_3x(b)+m_4x^1(b)=0$

With the assumption that at least one of m_1 and m_2 and one of m_3 and m_4 are non zero, if and only if $x(t)=\int_a^b G(s,t)f(s)ds$ **(12)**

b) Show that the boundary value problem $x^{11}+\cos x=0, x(0)=x(1)=0$ and show that this BVP has a unique solution. **(3)**

Unit-IV

15. a) Find the integral surface of the linear partial differential equation $x(y^2+z)p-y(x^2+z)q=(x^2-z^2)z$ which contains the straight line x+y=0, z=1. **(8)**

M.Sc. DEGREE EXAMINATION — MARCH/APRIL 2021

Part I – Previous

Branch – Mathematics

Paper IV – COMPUTING TECHNIQUES

(Revised Regulations from 2009–2010)

Max. Marks : 20

PART A

Answer any FOUR questions.

Each question carries 5 marks.

(Marks: 4 × 5 = 20)

1. How do you debug a shell script?

2. Discuss the cubic spline method for a BVP with an example.

3. What is meant by control of Statements? Explain in brief about any two types of control statements.

4. Define arrays. Explain about different types of arrays.

5. What are (a) Linear Fredholm integral equations of the 1st and 2nd (LFIE) kinds (b) Voltera integral equations of the 2nd kind (c) homogeneous and nonhomogeneous (LEIE) questions (d) eigen values and eigen functions (e) singular and nonsingular kernels.

6. What is meant by pointers? How they are used?

7. Write short notes on I/O multiplexing.

8. Discuss the essential aspects of Unix System which make it unique in design.

UNIT IV

15. Solve one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$; $y(0,t) = 0$; $y(l,t) = 0$, $y(x,0) = f(x)$ and $\frac{\partial y}{\partial t}(x,0) = 0 < x < l$.

Or

16. (a) Explain the cubic spline method in solving Fredholm integral equations.

(b) Solve $f(x) = \int_0^1 (x+t)\, f(t)dt = \frac{3}{2}x - \frac{5}{6}$.

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M.Sc. DEGREE EXAMINATION, APRIL/MAY -2022

PART I : PREVIOUS

BRANCH-MATHEMATICS

Paper - V : COMPLEX ANALYSIS

Revised Regulations from 2009-2010

(FOR DDE Students Only)

(Regular/Supplementary)

Max. Marks :20

SECTION - A

Answer any **Four** questions. Each question carries 5 Marks. (4×5=20)

1. Describe the construction of the stereographic projection. (5)

2. State and Prove Liouville Theorem. (5)

3. If f(z) is analytic and not identically zero in some domain D containing $z = z_o$ then prove that its zeroes are isolated. (5)

4. Discuss the pole singularities of the function $f(z) = \frac{z^{1/2} - 1}{z - 1}$ (5)

5. Evaluate $I = \frac{1}{2\pi i} \oint_{C_2} \left(\frac{3z+1}{z(z-1)^3} \right) dz$ where c_2 is the circle $|z| = 2$ (5)

6. Evaluate $h(x)$ where the Laplace transform of $h(x)$ is given by $\hat{H}(S) = \frac{1}{S(S^2+1)}$ (5)

7. State and Prove Schwarz symmetry principle. (5)

8. Find all bilinear transformations that map 0 and 1 to 0 and 1 respectively. (5)

SECTION - B

Answer **One** question from each Unit. All questions carry equal Marks. (4×15=60)

UNIT-I

9. a) Determine where f(z) is analytic when $f(z) = (x + \alpha y)^2 + 2i(x - \alpha y)$ for α real and constant. (7)

b) Assume that f(z) is analytic and not constant in a domain D of the complex z-plane. Suppose that $f'(z_o) = f''(z_o) = \ldots\ldots = f^{(n-1)}(z_o) = 0$, while $f^{(n)}(z_o) \neq 0,\ z_o \in D$. Then prove that the mapping $Z \to f(z)$ magnifies n times the angle between two intersecting differentiable arcs that meet at (z_o). (7)

(OR)

16. a) Show that the function $W = \int_0^z \left(\frac{dt}{(1-t^6)^{\frac{1}{3}}} \right)$ maps a regular hexagon into the unit circle.

(8)

b) Prove that a necessary and sufficient condition for a bilinear transformation to map the disk $|z| < 1$ onto $|w| < 1$ is that it be of the form $w = \beta \frac{z-\alpha}{\bar{\alpha}z - 1}$, $|\beta| = 1,\ |\alpha| < 1$.

(7)

UNIT III

13. State and prove Cauchy's residue theorem, using this, evaluate the integral $\int_0^\infty \frac{\sin x}{x} dx$.

Or

14. (a) State and prove Rouche's theorem.

(b) The quantity $\frac{z - z_1}{z - z_2}; \frac{z_3 - z_1}{z_3 - z_2}$, called the cross ratio of the four points z_1, z_2, z, z_3 (in that order) then show that it is invariant under every fractional linear transformation.

UNIT IV

15. (a) Define a Mobius Transformation, prove that a Mobius transformation takes circles onto circles.

(b) Show that Mobiues transformation $w = \frac{z-1}{z+1}$ maps the half-plane $\text{Re}\, z > 0$ onto the unit disc $|w| < 1$.

Or

16. Let $f : D \to \mathbb{C}$ be analytic and $f'(z) \neq 0$ for all $z \in D$, prove that f is conformal. Deduce that any bilinear transformation is conformal.